Oi, Try Judging a Math Olympiad — Day

OTJMO, 2021 ★ *Online Everywhere* ★★ *4 hours and 30 minutes* ★ *7 points per problem*

4. Call a positive integer m *cool* if it can be expressed as $7^x - 9^y$ for some positive integers x and y. Can the product or the sum of two cool integers ever be cool?

5. Triangle ABC, right angled at A, has circumcircle Γ. Point D on arc $\widehat{AB}$ and point E on arc $\widehat{AC}$ of Γ lie such that $AD = AE$. Lines BD and CE meet at K. The tangents to Γ at D and E meet at T. If the circumcircle of $\triangle DKT$ meets Γ again at M, and lines AB and KT meet at N, prove that the circumcenter of $\triangle BMN$ lies on KT.

6. Ivy reaches a magical world where she finds an infinite number of gift boxes, each having a different number of *bitcoins* inside them (i.e. at most a single box may be empty). She can choose k boxes, and she will receive all the bitcoins present in all those k boxes. Before she begins, she can randomly peek into k of the boxes and count the number of bitcoins in each of them. If Ivy can add properly, what maximum number of bitcoins (in terms of k) is she guaranteed to receive?